

# The Fundamentals of Stochastic processes

Sheet no.5

1) Medical research has shown that a certain type of chemotherapy is successful 70% of the time when used to treat skin cancer. Suppose five cancer patients are treated with this type of chemotherapy and let  $x$  equal the no. of successful cures out of the five.

$x$	0	1	2	3	4	5
$P(x)$	0.002	0.029	0.132	0.309	0.360	0.168

The probability distribution of  $x$  is given in the following table.

Find:

a)  $\mu = E(x)$

b)  $\sigma = \sqrt{E(x - \mu)^2}$

2) Find the expectation, variance and the standard deviation of each of the following:

i)

$x$	2	3	11
$P(x)$	$1/3$	$1/2$	$1/6$

ii)

$x$	-5	-4	1	2
$P(x)$	$1/4$	$1/8$	$1/2$	$1/8$

iii)

$x$	1	3	4	5
$P(x)$	0.4	0.1	0.2	0.3

iv)

$$p(x) = \begin{cases} \frac{2}{25}x & 0 \leq x \leq 5 \\ 0 & \text{elsewhere} \end{cases}$$

3) Prove for any random variable  $x$

i)  $E(ax+b) = aE(x) + b$

ii)  $V(ax+b) = a^2V(x)$

iii)  $E(c) = c$

iv)  $V(c) = 0$

4) The heart association claims that only 10% of adults over 30 can pass the physical fitness test. Suppose that four adults are randomly selected and each is given the fitness test.

a) Find the probability that <sup>none</sup> three of the four adults pass the test

b) Find the probability that three of the four adults pass the test

c) Let  $x$  represent the number of the four adults who pass the test

d) Drive a formula for  $p(x)$ , the probability distribution of the binomial random variable  $x$ .

5) Refer to problem 4. Use the formula for a binomial random variable to find the probability distribution of  $x$ , where  $x$  is the number of adults who pass the fitness test, graph the distribution

$x$	0	1	2	3	4
$P(x)$	0.6561	0.2916	0.0406	0.0036	0.0001

6) Refer to problem 5. Calculate the mean and the standard deviation.

7) Give a formula for  $p(x)$  for a binomial random variable with  $n=7$  and  $p=0.2$

8) Consider the following binomial probability distribution

$$P(x) = \binom{5}{x} (0.7)^x (0.3)^{5-x}, X = 0, 1, 2, 3, 4, 5$$

a) How many trials  $n$  are in the experiment?

b) What is the value of  $p$ , the probability of success?

c) Graph  $p(x)$

d) Find the mean and the standard deviation of  $x$ .

9) Suppose  $X$  is a binomial random variable with  $n = 3$  and  $p = 0.3$

a) Calculate the value of  $p(x)$ ,  $x=0, 1, 2, 3$ , using the formula for a binomial probability distribution.

b) Find the mean and the standard deviation of  $x$

10) If  $x$  is a binomial random variable. Calculate mean, variance and standard deviation for each of the following

a)  $n=80$  ,  $p=0.2$

b)  $n=70$  ,  $p=0.9$

c)  $n=1000$  ,  $p=0.04$